The spin-orbit interaction (SOI) allows electrical manipulation of individual spins and has therefore become a key ingredient for the realization of fully electrically controlled spin qubits [1,2]. For electrons in Si it is rather weak and synthetically boosted by means of micromagnets [3,4]. For holes, on the other hand, it is an intrinsic property which allows to perform electron dipole spin resonance (EDSR) measurements [1,2,5–9]. In Ge it is particularly strong, allowing for fast all electrical qubit manipulation, and yet an in-depth understanding of this interaction in hole systems is missing. Here we investigate, experimentally and theoretically, the effect of the cubic Rashba spin-orbit interaction on the mixing of the spin states by studying singlet-triplet oscillations in a planar Ge hole double quantum dot. Landau-Zener sweeps at different magnetic field directions allow us to disentangle the effects of the spin-orbit induced spin-flip term from those caused by strongly site-dependent and anisotropic quantum dot $g$ tensors. Our work, therefore, provides new insights into the hole spin-orbit interaction, necessary for optimizing future qubit experiments.

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sensitive to differences in, or the average of, the Zeeman energies of the dots, and hence we define $g^\pm = g^+ \pm g^-$ as the $g$-factor difference and sum. The energy spectrum of the system (the complete Hamiltonian $H_{0\text{d}}$ is derived in the Supplemental Material [17] Sec. VI) is depicted in Fig. 1(b) as a function of $\epsilon$. At $\epsilon = \epsilon^\ast$ the $S$ and $T_-$ states anticross.

We start by mapping out $\Delta_{ST}$ as a function of magnetic field angle by varying the magnetic field strength $B$ and $\epsilon$ [24]. We initialize the system deep in (2,0) in a singlet state [point I in Fig. 1(c)], then pulse quickly to (1,1) where the spins are separated (Sep). Mixing between $S$ and $T_-$ is induced when $\epsilon \approx \epsilon^\ast$. In the end we measure the spin state inside the PSB triangle ($M$). The resulting triplet return probability depends both on the size of the avoided crossing and the separation time $\tau_S$. We apply a rapid pulse of duration $\tau_R = 65 \text{ ns}$ and varying $\epsilon$ [inset of Fig. 1(d)]. Figures 1(d),1(e), and 1(f) depict the phase response of the charge sensor in the measurement point as a function of $\epsilon$ and $B$ for $\theta = 90^\circ$, $60^\circ$, and $10^\circ$, respectively. A high phase signal corresponds to a larger triplet return probability. In the out-of-plane direction we observe the expected funnel shape of the $S - T_-$ anticrossing [24]. At $60^\circ$ we similarly observe a typical funnel shape, however, we notice the line to be fainter, which indicates a smaller $\Delta_{ST}$. The picture drastically changes towards the in-plane direction where the $S - T_-$ avoided crossing develops interference fringes with a pattern resembling a butterfly; 2 components can be attributed to $S - T_-$ oscillations at low detuning and $S - T_0$ oscillations becoming more prominent at high detuning. The angular anisotropy of the funnel pattern, further exemplified in the Supplemental Material [17], Fig. S5, is the main focus of this work and requires knowledge of the full Hamiltonian and therefore an understanding of the interplay between the $g$-factor anisotropy and the spin-flip element $t_{SO}$.

In order to extract the $g$-factor anisotropy we rely on singlet-triplet oscillations. After initialization in $S(2,0)$, appropriate pulses to (1,1) induce either $S - T_0$ or $S - T_-$ oscillations. The probability to maintain the initial eigenstate of the system after a sweep with ramp time $\tau_R$ is given by the Landau-Zener formula $P_{LZ} = \exp[-(2\pi\Delta_{ST}^2/\hbar\nu)]$ [25,26], where $\hbar$ is the reduced Planck constant. $\nu = |dE/dt| = |dJ(e)/de|_{e=0}\Delta(e)/\tau_R$ is the velocity calculated at $\epsilon = \epsilon^\ast$ and $J(e) = (\sqrt{e^2/4} + 2\tau_R^2 - e/2)$ is the exchange energy [Fig. 1(b)] [15]. If $\nu$ satisfies the diabatic condition ($P_{LZ} \approx 1$) $S - T_0$ oscillations with a frequency $f = (1/\hbar)\sqrt{J^2 + (g^-\mu_B B)^2}$ will be favored. With $P_{LZ} < 1$ the $S - T_0$ oscillations are suppressed and the qubit is initialized in a superposition of $S$ and $T_-$. After a time $\tau_S$ the system is pulsed back to the measurement point where another nonadiabatic passage will cause an interference between the two states [27]. The accumulated phase difference is then given by $\phi = 2\pi f_{S,T-}\tau_S \approx (\tau_R/\hbar)|J - \frac{1}{2}g^+\mu_B B|$ [28] [Fig. 1(b)]. As the oscillation frequency of the $S - T_0$ ($S - T_-$) qubit depends on $g^-$ ($g^+$) [Fig. 1(b)] we can extract the individual $g$ factors without the need for EDSR. We fix the magnetic field at $|B| = 2 \text{ mT}$ and observe the oscillations vs $\tau_S$ while rotating $B$. We use a fast pulse ($\tau_R = \tau_{\text{rise}} = 2 \text{ ns}$) in Fig. 2(a) and a ramped pulse with ramp time $\tau_R = 100 \text{ ns}$ in Fig. 2(c). In both cases we pulse to $\epsilon = 4 \text{ meV}$ for a duration $\tau_R$. From the fast Fourier transform (FFT) in Figs. 2(b) and 2(d) we extract the oscillation frequency $f_{S,T_0}$ (orange dots) and $f_{S,T_-}$ (pink dots). We notice that for $\theta \in [-25^\circ, +25^\circ]$ in both FFT plots the $S - T_-$ frequency is visible, suggesting that a large coupling term inducing $S - T_-$ oscillations is present at these magnetic field directions, in line with the observations in Fig. 1(f). Moreover, in Fig. 2(d) the FFT power vanishes for $\theta \approx 60^\circ$ indicating that the ramp time $\tau_R$ induces a completely diabatic passage over the avoided crossing. This is in line with Fig. 1(e) where we observed a
sharper $S - T_-$ avoided-crossing characteristic of a smaller mixing term.

The lines arising in the FFT plots can be fit by the energy splitting between the three lowest lying states of the system depicted in Fig. 2(e) with $g^g = 12.00$, $g^R = 2.04$, $g^T = 0.10$, $g^J = 0.43$, and $t_C = 11.38 \mu$eV. The latter is extracted from exchange oscillation measurements (see Supplemental Material [17] Fig. S2).

Interestingly $|g^g| > |g^T|$ while $|g^T| < |g^J|$. This means that the $g$ factors in the out-of-plane direction have the same sign while they exhibit opposite signs in the in-plane direction. To understand this observation we investigate the effect of the dot geometry on the $g$ factors. As is shown in Supplemental Material [17] Sec. VIII by using the semimicroscopic Luttinger-Kohn Hamiltonian as a starting point, the effects of the intrinsic HH-LH mixing and an elliptical confinement potential can combine to yield $g$-factor renormalizations. While the correction to the out-of-plane $g$ factor is $|\delta g_j^g| < 10^{-2}$ for the values considered and hence negligible, the in-plane $g$ factor can be altered considerably,

$$g_j = \frac{1}{2} - \xi_j - \epsilon_j / \hbar (\omega_x - \omega_y) - \xi_2 \Delta.$$ \hspace{1cm} (1)

Here, $\xi_1 \approx 20.3$ and $\xi_2 \approx 6.0$ are material specific constants, $\Delta$ is the HH-LH splitting, $\hbar \omega_{x,y}$ are the in-plane confinement energies, and $g_j^0 = 0.2$ for Ge [30]. It can be seen from Fig. 2(f) that the in-plane $g$-factor corrections can be negative in one dot but not in the other for opposite elliptical confinement.

Electrostatic simulations of the DQD potentials arising from the applied gate voltages [29], not accounting for random disorder potentials, confirm the differently shaped dots. In fact, both dots appear elongated with the major axis of the dots being almost perpendicular to each other [Fig. 2(g) shows the calculated hole density $n_h$ and Supplemental Material [17] Sec. IX gives details about the simulation].

We now turn to extract $t_{SO}$ by analyzing $\Delta_{ST_\perp}$ in more detail. We perform Landau-Zener sweeps at $|B| = 20$ mT and extract $\Delta_{ST_\perp}$ from $P_{LZ}$ [Fig. 3(b)] and repeat this for different $\theta$. We vary $\tau_0$ during the first passage over the avoided crossing, creating a superposition of $S$ and $T_\perp$, and keep the return sweep diabatic in order to maintain this superposition [Fig. 3(a) and inset of Fig. 3(b)]. The extracted $\Delta_{ST_\perp}$ is reported for different $\theta$ in Fig. 3(c).

In general, $\Delta_{ST_\perp}$ may depend on effects influencing the hole spins such as the $g$-factor differences in the two dots, the SOI and possible effective magnetic field gradients caused by the hyperfine interaction [31]. While the hyperfine interaction can result in a strong out-of-plane hyperfine component $\delta b_z$ for HH states due to a special Ising-type form [32], the inhomogeneous dephasing times extracted for $B_z$ of $\approx 700$ ns at 1 mT in Ref. [15] give an upper limit for the hyperfine component $\delta b_z < 2$ neV, suggesting that the effects of the nuclear spin bath may safely be neglected.

In planar HH DQD systems the SOI can be parametrized by a real in-plane spin-orbit vector $t_{SO} = (t_x, t_y, 0)$. Such in-plane spin-flip tunneling terms stem from the cubic Rashba SOI [33], while this type of SOI does not induce out-of-plane terms $t_z$. In a basis in which the total Hamiltonian is diagonal in the absence of the SOI and $g$-factor differences, the $S - T_-$ splitting has the form [34]
FIG. 3. (a) Energy level diagram of the states involved in the passage over the avoided crossing [red circle in 1(b)]. The probability $P_S$ to maintain a singlet after a single passage over the avoided crossing is given by the Landau-Zener formula. (b) The single LZ passage pulse sequence (inset) leads to a singlet return probability $P_S$ that decays exponentially with the ramp time $\tau_R$. A fit to the Landau-Zener transition formula (black dashed line) allows to extract $\Delta_{ST_\perp}$. (c) $\Delta_{ST_\perp}$ as a function of magnetic field angle. The extracted $\Delta_{ST_\perp}$ is fit to Eq. (2) with $t_c$ and $t_s$ as fitting parameters (solid blue line). The black dashed line represents the maximum $\Delta_{ST_\perp}$ as a function of $\theta$ that can be reliably measured by a single LZ passage. The light colored data points are, therefore, excluded from the fit. (inset) Comparison between the analytical result [solid line, Eq. (2)] and numerical simulation (squares) for $\theta = 90^\circ$. The light colored data points are, therefore, excluded from the fit. (d) Comparison between the two contributions to $\Delta_{ST_\perp}$.

\[
\Delta_{ST_\perp} = \left| \Delta_{SO} \sin \left( \frac{\Omega}{2} \right) + \Delta_{E_Z} \cos \left( \frac{\Omega}{2} \right) \right|, \tag{2}
\]

where the spin-orbit splitting $\Delta_{SO}$ and the Zeeman splitting $\Delta_{E_Z}$ due to anisotropic site-dependent $g$ tensors read

\[
\Delta_{SO} = t_s + i t_c \frac{g_\perp^1 \sin \theta}{\sqrt{(g_\perp^1 \cos \theta)^2 + (g_\perp^1 \sin \theta)^2}}, \tag{3}
\]

\[
\Delta_{E_Z} = \frac{\mu_B}{4\sqrt{2}} \frac{(g_\parallel^1 g_\perp^1 - g_\perp^1 g_\parallel^1) \sin(2\theta)}{\sqrt{(g_\parallel^1 \cos \theta)^2 + (g_\perp^1 \sin \theta)^2}}, \tag{4}
\]

and $\Omega = \arctan(2\sqrt{2} t_c/e^*)$ is the mixing angle at the anticrossing. The analytical result (2) agrees well with the numerical results obtained by exact diagonalization of the system Hamiltonian for all $\theta$ except in a narrow region around $\theta = 0$ [\textit{inset}, Fig. 3(c)]. We attribute these deviations to the small in-plane Zeeman energies which violate the assumption of an isolated two-level system made when deriving (2) (see Supplemental Material [17 Sec. VI]). Because of the opposite sign $g$-factor corrections in the dots the Zeeman splitting $\Delta E_Z$ can be the dominant contribution to $\Delta_{ST_\perp}$, exceeding the spin-orbit splitting by one order of magnitude at small angles. Even when the magnetic field has a large out-of-plane component, the effect of different $g$ factors can contribute crucially to $\Delta_{ST_\perp}$ [Fig. 3(d)].

The extracted $\Delta_{ST_\perp}$ in Fig. 3(c) can be fit by the model with $t_c$ and $t_s$ as free parameters and $t_c$, $g_{\parallel}^1$, $g_{\perp}^1$, $g_{\parallel}^1$, $g_{\perp}^1$ extracted from previous measurements. Between $-25^\circ$ and $25^\circ$ the splitting seems to drop to zero as the Landau-Zener assumptions of diabatic return sweeps are not met and an extraction of $\Delta_{ST_\perp}$ is not accurate. The black dashed line corresponds to the maximum $\Delta_{ST_\perp}$ that allows a diabatic passage with a rise time of 2 ns of our pulses ($P_{LZ,\text{max}} = 0.99 = \exp(-2\pi \Delta_{ST,\text{max}}^2 / \hbar \tau)$). The model fits the dark blue data points with $t_c = 129.0 \pm 18.0$ and $t_s = -369.8 \pm 13.8$ neV, yielding the total spin-flip tunneling element $t_{SO} = \sqrt{t_c^2 + t_s^2} = 392.0$ neV.

Having characterized all the elements in the Hamiltonian from independent measurements we can now reproduce the funnel measurements in Fig. 1 (Fig. 4). In particular the sharper line at $\theta = 60^\circ$ [Fig. 4(b)] as well as the $S-T_\perp$ oscillations for $\theta = 10^\circ$ [Fig. 4(c)] reflect what we observe in the data. Even with $t_{SO} = 0$ the in-plane $g$-factor difference induces $S-T_\perp$ oscillations [Fig. 4(d)] further confirming its dominant role in determining the size of $\Delta_{ST_\perp}$.

In conclusion, we have demonstrated that the $g$-tensor anisotropy and, in particular, the in-plane $g$-factor
difference can lead to a considerable contribution to $\Delta_{ST,rot}$ in the in-plane direction. However, Landau-Zener sweeps and singlet-triplet oscillations measured in different magnetic field directions allowed us to distinguish the Zeeman induced coupling from the spin-orbit induced coupling and, thereby, infer the magnitude and orientation of $t_{SO}$. We reconstructed the experimental data in our simulations confirming the validity of our theoretical model. This understanding of the interplay between $t_{SO}$ and the in plane $g$-factor difference opens the possibility to operate hole singlet-triplet qubits at sweet spots, for example with orthogonal axis [34]. Our work, therefore, provides important insight into the spin-orbit interaction of hole spin double quantum dot devices and lays the foundation for the design of future hole spin qubit experiments.

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